In the case of a survey such as mine, it is not clear that blindness to differences in T and IE physics instruction could have been found in any potential surveyors, physics teachers, or students who were not medically institutionalized. Non-double-blind education research experiments may be less convincing than some double-blind medical experiments, but that doesn’t mean that the education results should necessarily be taken at less than face value. In the case of my survey, I think that the results merit acceptance at full face value, especially considering the fact that normalized gain differences between T and IE courses that are consistent with those I reported have now been obtained by physics education research groups at the University of Maryland [Redish et al. (1997), Saul (1998), Redish and Steinberg (1999), Redish (1999)], the University of Montana [Francis et al. (1998)], Rensselaer and Tufts Universities [Cummings et al. (1999)], North Carolina State University [Beichner et al. (1999)], Hogskolan Dalarna—Sweden [Bernhard (2001)], Carnegie Mellon University [Johnson (2001)], and City College of New York [Steinberg and Donnelly (2002)].

4. “I would be more willing to accept that IE classes show greater conceptual gains if those gains were shown to be of value in follow-on courses, specifically if they led not only to greater student entré in the major, but more importantly to higher numbers of physics graduates.”

This is a legitimate reason for doubt, but already some longitudinal studies have shown encouraging results that may reduce Ehrlich’s skepticism: for example, evidence (see Ref. 3 for references) that learning in IE physics courses is substantially retained 1 to 3 years after the courses have ended [Chabay (1997), Francis et al. (1998), Bernhard (2001)]. But increasing the numbers of physics graduates will require, in my view, in addition to increased effectiveness of introductory courses, a move toward “science/math literacy for all” in K–12 education,4 and the implementation of the 1960’s Curriculum S for “Synthesis.”5


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Zarmi’s recent paper on Bertrand’s theorem,1 like most of the others of which we are aware, relies on power expansions to prove it. We want to point out that besides the ones explicitly mentioned in Zarmi’s paper, there are other noteworthy proofs deserving to be mentioned explicitly.

The proof in Ref. 2 uses classical techniques but is more pedagogical in character than Bertrand’s original treatment and analyzes various special cases to first establish and then clarify the fact that not every closed orbit in a central potential needs to be elliptical. We believe that this proof was intended to be a centennial celebration of Bertrand’s theorem.

The work in Ref. 3 uses an expansion involving the inverse of the radial coordinate and phase space techniques for proving Bertrand’s theorem. Their method can be described as a translation into a physicist’s language of Arnold’s proof of the theorem which is given as a set of problems in his well-known book.4

Then there is our proof that does not have to rely too much on local properties of the orbits nor has to start from a series expansion.5,6 The demonstration takes advantage of the additional constants of motion7,8 that the existence of periodic plane orbits necessarily implies.4,5 Because the orbits are closed, an additional constant—in addition to the energy and the angular momentum—is needed to pin down the orbits and preventing them from wandering all over the orbital plane. We then show that this additional constant can only be a vector lying on the orbital plane or a constant matrix whose eigenvectors should also lie on such a plane. The exact form of the extra constant depends on how many dynamical symmetry axes the orbits may have. The symmetry properties of the extra constant determine the functional form of the potential producing the periodic orbits. A vector constant is possible only if the potential is the Newtonian one, \( V_N(r) = -a/r \). The extra constant is the Laplace–Runge–Lenz vector in this case. A matrix constant is possible only if the
potential is the harmonic oscillator one, \( V_{ho}(r) = kr^2/2 \), and in this case the first integral is the constant matrix \( A_{ab} = p_\alpha p_\beta/2 + k x_\alpha x_\beta r^2 (a, b = 1, 2, 3) \). One of the eigenvectors of \( A_{ab} \) is parallel to the angular momentum \( L \) and its corresponding eigenvalue vanishes. This feature of its eigenvector and its corresponding eigenvalue means that the matrix is orthogonal to the orbital plane. The eigenvectors corresponding to the nonvanishing eigenvalues define the two dynamical symmetry axes of the harmonic oscillator orbits. This proof illustrates in yet another way the power of symmetry in classical mechanics.